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BOILING MODEL FOR A FLUIDIZED BED OF PARTICLES

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A mechanism for boiling of a fluidized bed was examined in [1]. Due to hydrodynamic instability the solid particles acquire random motion, and as a result of collisions between particles part of the energy of random motion is converted to rotation of the particles. A rotating particle experiences a Magnus force which considerably increases the random motion and leads to spontaneous boiling of the layer. For this mechanism there is typically a minimum boiling time τ_γ, defined basically as the time to develop a hydrodynamic instability. It is shown in this study that besides the spontaneous mechanism there is an induced mechanism for boiling of the bed arising from the generation of random motion in one particle layer. Particles in that layer boil, transmitting a perturbation to the energy of the next layer, and leading to layer boiling in a manner analogous to the propagation of a detonation wave in solids.

1. We consider a bed of rather densely packed spherical particles at rest, supported on a grating permeable to gas, through which gas is circulated from below. When a certain gas velocity is reached the particles become "weightless", i.e., the gravity force becomes equal to the drag force. Such a bed of gas and particles is conventionally called fluidized. However, this state of the bed is unstable, and after a certain time the bed boils. The behavior of the particles in a boiling bed is reminiscent of that of gas molecules, and therefore by analogy we shall call them a gas of particles. The system of equations describing the motion of the mixture, allowing for the Magnus force, as given in [2], and in the notation adopted in [3], has the form

$$\frac{\partial\rho_1}{\partial t} + \nabla(\rho_1\mathbf{v}_1) = 0, \quad \rho_1 = \rho_{11}m_1,$$
$$\frac{\partial\rho_2}{\partial t} + \nabla(\rho_2\mathbf{v}_2) = 0, \quad \rho_2 = \rho_{22}m_2,$$
$$\rho_1\frac{d_1\mathbf{v}_1}{dt} = -m_1\nabla p_1 - \mathbf{f}_{12}, \quad \frac{d_1(\ )}{dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{v}_1\cdot\nabla),$$
$$\rho_2\frac{d_2\mathbf{v}_2}{dt} = -m_2\nabla p_1 + \mathbf{f}_{12} - \nabla p_2 + \mathbf{g}\rho_2,$$
$$\frac{d_1e_1}{dt} + p_1\frac{d_1}{dt}\left(\frac{1}{\rho_{11}}\right) = \mathbf{f}_{12}(\mathbf{v}_1 - \mathbf{v}_2)\Big/\rho_1 - \frac{q_{12}}{\rho_1} - \frac{Q_M}{\rho_1},$$
$$\frac{d_2e_2}{dt} = \frac{\dot q_{12}}{\rho_2} + \frac{Q_D}{\rho_2}, \quad \dot q_{12} = \varkappa(T_1 - T_2), \quad e_2 = c_2T_2,$$
$$\frac{d_2(\ )}{dt} = \frac{d(\ )}{\partial t} + (\mathbf{v}_2\cdot\nabla), \quad e_1 = c_1T_1, \quad \varepsilon_2 = 3c^2,$$
$$\frac{d_2\varepsilon_2}{dt} = -p_2\frac{d_2}{dt}\left(\frac{1}{\rho_2}\right) + \frac{\dot Q}{\rho_2}, \quad \psi = 1 - 1{,}17m_2^{2/3}, \tag{1.1}$$

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$$\mathbf{f}_{12} = \frac{3}{4}\frac{c_d}{d} m_2\rho_{11}\frac{|\mathbf{v}_1-\mathbf{v}_2|(\mathbf{v}_1-\mathbf{v}_2)}{\psi^2},$$

$$\dot{Q} = Q_M - Q_D, \quad Q_M = \sqrt{30}\,\rho_{11}m_2c^2|\mathbf{v}_1-\mathbf{v}_2|/d,$$

$$Q_D = \sigma\rho_{22}c^3\varphi^0(m_2)/d, \quad \varphi^0(m_2) = m_2\Bigg/\left(\left(\frac{m_2^*}{m_2}\right)^{1/3}-1\right),$$

$$p_2 = m_2\rho_{22}c^2/\eta, \quad \eta = 1-\left(\frac{m_2}{m_2^*}\right)^{1/3}, \quad \sigma = \frac{\sqrt{3}}{2}\left(\frac{1}{7}+\frac{1-k^2}{2}\right),$$

$$p_1 = \rho_{11}RT_1, \quad m_1+m_2=1, \quad \rho_{22}=\text{const}, \quad l/d = \left(\frac{m_2^*}{m_2}\right)^{1/3}-1,$$

where ρ_{11} is the gas density; m_1 and m_2 are the volume content of the phases; ρ_{22} is the density of the particle material; p_1 is the gas pressure; p_2 is the particle gas pressure; T_1 is the gas temperature; c is the random particle velocity; T_2 is the particle temperature; $\mathbf{v}_1$ is the gas velocity; $\mathbf{v}_2$ is the particle velocity; $\mathbf{f}_{12}$ is the interphase interaction force per unit volume; e_1 is the specific energy of the gas; e_2 is the specific internal energy of the particles; ε_2 is the specific energy of the gas of random motion of the particles; $\dot{q}_{12}$ is the heat flux from the gas to the particles per unit time; $\dot{Q}$ is the rate of transfer of energy from the gas to energy of random motion of the particles per unit volume; Q_M is the work done by Magnus force; Q_D is the rate of dissipation of energy of random motion into thermal energy on particle collisions; k is the coefficient of momentum restitution for a frontal collision of particles, assumed equal to 1 below (k = 1), corresponding to elastic collisions of the particles; m_2^* is the volume content of the solid phase with the particles densely packed; l is the distance between particles; d is the particle diameter; and c_d is the drag coefficient. Since $\text{Re} = |\mathbf{v}_1 - \mathbf{v}_2|d/\nu \gg 1$ (ν is the kinematic viscosity) we may choose $c_d \simeq 1/2$. Below we shall denote the mixture parameters for dense particle packing by the subscript $*$. Thus, the problem is to determine the rate of boiling of the bed D in the region $\Omega\langle 0\leqslant x\leqslant L, 0\leqslant t\leqslant T\rangle$ (Fig. 1). Here L is the bed thickness, T is the bed boiling time for fixed gas flow and particle parameters satisfying the steady-state system of equations and the given constant parameters of the gas flow at the bed inlet ($j = \tilde{j}$, $v_1 = \tilde{v}_1$, $p_1 = \tilde{p}_1$, $T_1 = \tilde{T}_1$) and for fixed bed porosity $m_2|_{x=0} = \tilde{m}_2$ (j is the gas flow rate). The coordinate system is chosen so that the direction of motion of the gas in Ω is positive, and the direction of action of the gravity forces is negative ($\mathbf{g} = -g\mathbf{e}_x$).

Assuming in the equations of the system (1.1) that all the partial derivatives are zero, and $\mathbf{v}_2 = 0$, $c = 0$, and neglecting the heat transfer between the gas and the particles and the variation of gas temperature, we obtain a system of ordinary differential equations describing the steady state of the system in the form

$$\frac{d}{dx}(\rho_1 v_1) = 0, \quad \rho_1 v_1\frac{dv_1}{dx} = -m_1\frac{dp_1}{dx} - f_{12}, \quad m_1+m_2 = 1, \tag{1.2}$$

$$f_{12} - m_2\frac{dp_1}{dx} - g\rho_2 = 0, \quad \rho_1 = \rho_{11}m_1, \quad \rho_2 = \rho_{22}m_2, \quad T_1 = \tilde{T}_1,$$

$$p_1 = \rho_{11}R\tilde{T}_1, \quad f_{12} = (3/4)\,c_d m_2\rho_{11}v_1^2/d$$

with the initial conditions

$$(\rho_{11}, v_1, m_1)|_{x=0} = (\tilde{\rho}_{11}, \tilde{v}_1, \tilde{m}_1).$$

Reducing the system of equations obtained (1.2) to the normal form, we have

$$\frac{dv_1}{dx} = -\frac{1}{\tilde{j}}\left(\frac{c_x\rho_{11}v_1^2}{\psi^2} - g\rho_{22}m_1\right), \quad \tilde{j} = m_1\rho_{11}v_1,$$

$$\frac{d\rho_{11}}{dx} = \frac{1}{R\tilde{T}_1}\left(\frac{c_x\rho_{11}v_1^2}{\psi^2} - g\rho_{22}\right), \quad c_x = (3/4)\,c_d/d,$$

and, dividing the first equation by the second, we obtain

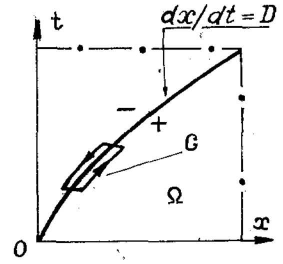

Fig. 1

$$\frac{dv_1}{d\rho_{11}} = -\frac{R\widetilde{T}_1}{\widetilde{j}} \frac{\left(c_x\rho_{11}v_1^2 - \frac{g\rho_{22}}{\rho_{11}v_1}\dot{\widetilde{j}}\psi^2\right)}{\left(c_x\rho_{11}v_1^2 - g\rho_{22}\psi^2\right)}.$$

This equation can be integrated in the region $0 \leqslant x \leqslant L$, since it has no singularities anywhere in the region ($m_1 < 1$). The solution of the equation will be a relation $v_1 = v_1(\rho_{11}, A)$ (A is a constant of integration), and by substituting this into the second equation and allowing for the initial conditions we obtain $v_1^0(x)$, $\rho_{11}^0(x)$, $m_1^0(x)$, $p_1^0(x)$; here the quantities with superscript zero refer to the zero-order approximation.

2. We now consider the matter of propagation of a boiling wave through the particle gas. For the particles this wave is "strong", and we shall therefore use the full system of equations for the second component of Eq. (1.1). For the gas we use the linearized equations for perturbations, which we obtained by taking account of the steady state as a zero-order approximation. We represent the desired solution in the form

$$\begin{gathered} v_1 = v_1^0(x) + v_1^1(x,t), \quad p_1 = p_1^0(x) + p_1^1(x,t), \\ \rho_1 = \rho_1^0(x) + \rho_1^1(x,t), \quad m_1 = m_1^0(x) + m_1^1(x,t), \\ T_1 = \widetilde{T}_1 \ (T_1^1/\widetilde{T}_1 \ll 1), \quad v_2 = v_2(x,t), \ p_2 = p_2(x,t). \end{gathered} \tag{2.1}$$

Substituting Eq. (2.1) into system (1.1), neglecting effects of heat transfer between the gas and the particles, and taking account of Eq. (1.2), we obtain the equations

$$\begin{gathered} \frac{\partial}{\partial t}(\rho_1^1) + \frac{\partial}{\partial x}(\rho_1^0 v_1^1 + \rho_1^1 v_1^0) = 0, \\ \rho_1^0 \frac{\partial}{\partial t}(v_1^1) + (\rho_1^0 v_1^1 + \rho_1^1 v_1^0)\frac{\partial v_1^0}{\partial x} + \rho_1^0 v_1^0 \frac{\partial v_1^1}{\partial x} = -m_1^0 \frac{\partial p_1^1}{\partial x} - \\ - m_1^1 \frac{\partial p_1^0}{\partial x} - f_{12}^1, \quad \rho_1^1 = \rho_{11}^1 m_1^0 + \rho_{11}^0 m_1^1, \\ \frac{\partial}{\partial t}(\rho_2) + \frac{\partial}{\partial x}(\rho_2 v_2) = 0, \quad \frac{\partial}{\partial t}(\rho_2 v_2) + \frac{\partial}{\partial x}(\rho_2 v_2^2 + p_2 + m_2^0 p_1^1) = G, \\ \frac{\partial}{\partial t}\left(\rho_2\left(\varepsilon_2 + \frac{v_2^2}{2}\right)\right) + \frac{\partial}{\partial x}\left(\rho_2 v_2\left(J_2 + \frac{v_2^2}{2}\right)\right) = \frac{\dot{Q}}{\rho_2} - Gv_2 - v_2\frac{\partial}{\partial x}(m_2^0 p_1^1), \\ J_2 = \varepsilon_2 + p_2/\rho_2, \quad m_1^1 = -m_2^1, \\ G = f_{12}^1 - g\rho_2^1 - m_2^1\frac{dp_1^0}{dx} + p_1^1\frac{dm_2^0}{dx}, \end{gathered} \tag{2.2}$$

where

$$\left|\frac{p_1^1}{p_1^0}\right| \ll 1, \quad \left|\frac{p_1^1}{p_2}\right| < 1, \quad \left|\frac{G}{\frac{\partial p_2}{\partial x}}\right| < 1, \quad \left|\frac{m_1^1}{m_1^0}\right| \ll 1, \quad \left|\frac{v_1^1}{v_1^0}\right| \ll 1. \tag{2.3}$$

The inequalities (2.3) hold for the actual parameters ρ_{11}^0/ρ_{22}, $\widetilde{T}_1$, m_2,

$$\frac{\rho_{11}^0}{\rho_{22}} > 3\frac{m_2^0}{m_1^0}10^{-3}, \quad \left|\frac{m_2^0 - m_2^*}{m_2^*}\right| \ll 1, \quad (\tilde{v}_1)^2/R\tilde{T}_1 \ll 1. \tag{2.4}$$

After the particles are excited at the front of the wave the Magnus force becomes active ($\dot{Q} > 0$) which compensates for the particle energy loss ε_2 on collisions. An estimate of the energy release rate shows that the energy release occurs in a narrow zone of thickness $\Delta x \sim 8d$, and we therefore consider the wave front as a discontinuity. By analogy with [4] we go from the differential equations (2.2) to the integral laws of conservation, and choose the integration contour G (see Fig. 1) as in [4]. Using the inequality (2.3) we put $[\rho_{11}^1] = 0$ and, omitting terms of higher order of differentiability, we obtain

$$[p_1^1] = \frac{\rho_1^0(v_1^0 - D)^2}{m_1^0}[m_1^1], \quad [v_1^1] = -\frac{[m_1^1]}{m_1^0}(v_1^0 - D), \tag{2.5}$$

$$[\rho_{11}^1] = 0, \quad [\rho_2(v_2 - D)] = 0, \quad \left[J_2 + \frac{(v_2 - D)^2}{2}\right] = 0,$$

$$[\rho_2(v_2 - D)^2 + p_3] = 0, \quad p_3 = p_2(\varepsilon_2, m_2) + m_2^0 p_1^1,$$

where the square brackets denote the jumps of the appropriate quantities at the discontinuity; and $D = dx/dt$ is the wave velocity. The first two relations of system (2.5) coincide to an accuracy of $o((m_1^1)^2)$ with the expressions obtained in [5] on the basis of a model equation describing motion of a gas through a porous layer. As boundary conditions ahead of the discontinuity we take $\varphi = (\varepsilon_2, p_2) = 0$. Behind the wave front the condition

$$Q = 0 \tag{2.6}$$

holds, which defines the relation $\varepsilon_2 = \varepsilon_2(m_2)$ and closes the system (2.5). The system of equations obtained, (2.5) and (2.6), has an exact solution. Using the second inequality in Eq. (2.4), we obtain the relation $\left(\frac{m_2^*}{m_2}\right)^{1/3} - 1 = \frac{1}{3}\frac{\theta}{\vartheta_2^*} + o\left(\frac{\theta}{\vartheta_2^*}\right)^2$. We rewrite the equation of state and the particle gas enthalpy equation in the form $p_2 = \varepsilon_2/\theta$, $J_2 = p_2(\vartheta_2^* + 2\theta)$, where $\theta = \vartheta_2 - \vartheta_2^*$, $\vartheta_2 = 1/(\rho_{22}m_2)$, $\vartheta_2^* = 1/(\rho_{22}m_2^*)$, $\theta/\vartheta_2^* \ll 1$. Since the state is unperturbed ahead of the shock wave, we have $\theta^+ = \theta^0$, and by substituting $J_2(\theta)$ into the system (2.5), we obtain, to an accuracy of $o((\theta/\vartheta_2^*)^2)$

$$\frac{\Delta v_2}{D} = \frac{\theta^0 - \theta}{\vartheta_2^*}, \quad p_2 = \tilde{D}^2\frac{(\theta^0 - \theta)}{(\vartheta_2^*)^2}, \quad p_2 = p_2^+\left(\frac{3\theta^0 - \theta}{3\theta - \theta^0}\right), \tag{2.7}$$

$$p_1^1 = -\frac{\rho_{11}^0}{\rho_{22}}\frac{(v_1^0 - D)^2}{m_1^0}\frac{(\theta^0 - \theta)}{(\vartheta_2^*)^2}, \quad v_1^1 = \frac{(v_1^0 - D)}{m_1^0}\frac{\theta^0 - \theta}{\vartheta_2^*},$$

$$\tilde{D}^2 = D^2 + \frac{m_2^0}{m_1^0}\frac{\rho_{11}^0}{\rho_{22}}(v_1^0 - D)^2, \quad p_2^+ = 0, \quad \Delta v_2 = v_2^- - v_2^+.$$

The condition (2.6) gives $c = 14\frac{\rho_{11}^0}{\rho_{22}}\frac{\theta v_1^0}{\vartheta_2^*}$ or, allowing for the equation of state,

$$p_2 = 3\cdot(14)^2\left(\frac{\rho_{11}^0}{\rho_{22}}\frac{v_1^0}{\vartheta_2^*}\right)^2\theta, \tag{2.8}$$

and here p_2^+ is the pressure and θ^+ is the difference of the specific volume from ϑ_2^* ahead of the shock wave and the remaining quantities describe the state behind the shock wave. We shall study the process of boiling of the particle gas in the (p_2, θ) plane (Fig. 2).

3. In accordance with the third equation of the system (2.7) (Hugoniot adiabat) and $p_2^+ = 0$ we find that the state behind the shock wave lies on the vertical straight line passing through the point C, at some point B. The transition from the initial state A to B occurs in a jump and is described by the second equation of (2.7) (the law of conservation of momentum). The tangent of the angle of inclination of AB to the abscissa axis defines the shock wave

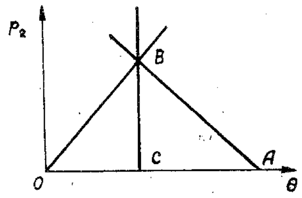

Fig. 2

velocity, and is found in turn by the intersection of the straight lines AB and OB (the line OB is described by Eq. (2.8)). Thus, the point of intersection of the lines AB, OB and CB determines the desired solutions:

$$\theta = \frac{\theta^0}{3}, \quad p_2 = (14)^2 \left(\frac{\rho_{11}^0}{\rho_{22}} \frac{v_1^0}{\vartheta_2^*} \right)^2 \theta^0.$$

Using the second equation of the system (2.7), we obtain

$$\tilde{D}^2 = \frac{3}{2} \left(14 \frac{\rho_{11}^0}{\rho_{22}} v_1^0 \right)^2,$$

which gives

$$D_{1,2} = \left(\frac{m_2^0}{m_1^0} \frac{\rho_{11}^0}{\rho_{22}} \pm \sqrt{\frac{3}{2} \left(14 \frac{\rho_{11}^0}{\rho_{22}} \right)^2 - \frac{\rho_{11}^0}{\rho_{22}} \frac{m_2^0}{m_1^0}} \right) v_1^0, \tag{3.1}$$

where $D_1 > 0$ corresponds to the wave propagating in the positive direction (direction of the gas flow), and $D_2 < 0$ corresponds to the wave propagating in the negative direction. The condition (2.4) on the parameter (ρ_{11}^0/ρ_{22}) ensures that the expression under the root sign is positive. We note that $D_1 > |D_2|$. The reason is that the stagnating action of the perturbed gas flow on the particles at the wave front D_1 is less than for the wave D_2:

$$[p_1^1]^+ = \frac{\rho_{11}^0 (v_1^0 - D_1)^2}{m_1^0} [m_1^1], \quad [p_1^1]^- = \frac{\rho_{11}^0 (v_1^0 + |D_2|)^2}{m_1^0} [m_1^1],$$

and hence, allowing for $[m_1^1] < 0$, we obtain $[p_1^1]^- < [p_1^1]^+ < 0$. Another special feature of the solution obtained for Eq. (2.7), i.e., Eq. (3.1), is the relation

$$\frac{v_2}{D} \approx \frac{2}{3} \frac{\theta^0}{\vartheta_2^*} = \frac{2l}{d} + o\left(\frac{\theta^0}{\vartheta_2^{*2}} \right) \ll 1. \tag{3.2}$$

This fact follows from the assumption that the particles are incompressible, and therefore the velocity of transmission of a perturbation in a particle is infinite. It can be seen that for a displacement of the particle by a mean free path l in time τ the perturbation is transmitted a distance $l + d$, and therefore, taking into account that $l \ll d$ we have $v_2/D = l/(l + d) \approx l/d$, which coincides with Eq. (3.2) to within a factor of two. Thus, on the basis of our solution of Eq. (2.7), i.e., Eq. (3.1), we can assert that there is an induced mechanism for boiling of the bed.

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